

Radicals

You can think about radicals (also called “roots”) as the opposite of exponents.

We already know that the expression x^2 with the exponent of 2 means “multiply x by itself two times”. The opposite operation would be “what do we have to multiply by itself two times in order to get x ?” That’s where radicals come in. If you see

$$\sqrt{x}$$

it means “the number you have to multiply by itself to get x .” The symbol that contains the x is called the “radical sign,” and the expression inside the symbol - in this case x - is called the “radicand.” Instead of saying that an expression is *inside* the radical sign, however, we usually say that it’s *under* the radical sign. When you see $\sqrt{x}$, you can call it “the square root of x .” But there are other kinds of roots of x too (which are indicated by little numbers tucked into the left side of the radical sign):

$$\sqrt[3]{x}$$

“the cube root of x ”

$$\sqrt[4]{x}$$

“the fourth root of x ”

$$\sqrt[5]{x}$$

“the fifth root of x ”

“The cube root of x ” means the number that’s multiplied by itself three times in order to get x ; “the fourth root of x ” means the number that’s multiplied by itself four times in order to get x , and so on.



Since roots are the opposite operation of exponents, you can convert between roots and exponents. For example, taking the square root of x is the same as raising x to the $1/2$ power. To see this, apply the power rule for exponents:

$$\left(x^{\frac{1}{2}}\right)^2 = x^{\left(\frac{1}{2} \cdot 2\right)} = x^1 = x$$

Here's how to convert between roots and exponents.

$$\sqrt{x} \quad \text{is the same as} \quad x^{\frac{1}{2}}$$

$$\sqrt[3]{x} \quad \text{is the same as} \quad x^{\frac{1}{3}}$$

$$\sqrt[4]{x} \quad \text{is the same as} \quad x^{\frac{1}{4}}$$

$$\sqrt[5]{x} \quad \text{is the same as} \quad x^{\frac{1}{5}}$$

Example

Simplify the radical expression.

$$\sqrt{9}$$

We're taking the square root of 9, which means we need to figure out what number we have to multiply by itself in order to get 9.

If we multiply 3 by itself, we get 9, which means that the square root of 9 is 3. So we can say

$$\sqrt{9} = 3$$



If we're given a number that x stands for, we want $\sqrt{x}$ to represent only one number - and a number that everyone will agree on. Notice, however, that we can get 9 not only by multiplying 3 by itself, but also by multiplying -3 by itself:

$$(+3)(+3) = 9 = (-3)(-3)$$

The way we get around this is that everyone agrees that both $\sqrt{9}$ and $9^{\frac{1}{2}}$ mean the positive number that you can multiply by itself in order to get 9:

$$\sqrt{9} = 3 = 9^{\frac{1}{2}}$$

Also

$$\sqrt{0} = 0 = 0^{\frac{1}{2}}$$

because 0 is the only number that when multiplied by itself gives 0. Now notice that there is no negative number that when multiplied by itself gives a negative number. (A positive number multiplied by itself is positive, 0 multiplied by itself is 0, and a negative number multiplied by itself is positive.) So $\sqrt{x}$ and $x^{\frac{1}{2}}$ are undefined if x is negative.

If x stands for any positive number, there's one and only one positive number that when multiplied by itself gives x . So $\sqrt{x}$ and $x^{\frac{1}{2}}$ are defined, and they represent that "one and only one positive number."

In later sections, we'll look at the specific rules we use to handle radical expressions.

